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The mean drift force and yaw moment on marine structures in waves and current

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The effect of the steady second-order velocities on the drift forces and moments acting on marine structures in waves and a (small) current is considered. The second-order velocities are found to arise due to first-order evanescent modes and linear body responses. Their contributions to the horizontal drift forces and yaw moment, obtained by pressure integration at the body, and to the yaw drift moment, obtained by integrating the angular momentum flux in the far field, are expressed entirely in terms of the linear first-order solution. The second-order velocities may considerably increase the forward speed part of the mean yaw moment on realistic marine structures, with the most important contribution occurring where the wave spectrum often has its maximal value. The contribution to the horizontal forces obtained by pressure integration is, however, always found to be small. The horizontal drift forces obtained by the linear momentum flux in the far field are independent of the second-order velocities, provided that there is no velocity circulation in the fluid.

1. Introduction

One of the important problems in fluid dynamics is the interaction between water waves and floating or submerged bodies. Practical examples are represented by freely moving or moored ships and oil platforms. The linear wave forces, being proportional to the wave amplitudes, are usually responsible for the major part of the loading, but the steady drift forces, being quadratic in the wave amplitudes, can also be very important, giving rise to a drift of the body. There is also a similar steady second-order moment in the vertical direction, the steady yaw moment, which is responsible for an angular drift in the horizontal plane.

The body often has a forward velocity which we shall here assume to be steady. Observed from the body, this forward velocity is equivalent to a uniform current in the opposite direction to the forward speed. Frequently, also a current is observed in the sea. In the North Sea, for instance, the current may be up to 1 ms^{-1}. We shall assume that this current may be approximated as uniform (in space and time) whereby the effects of a forward velocity and a current become mathematically identical.

One important reason for studying the forces on a floating body with a steady forward speed is the observed pseudo-steady resonant horizontal motions of moored oil platforms or ships due to nonlinear wave forces. It is reasonable that for sufficiently low-frequency and large-amplitude excursions this motion may be treated as steady in the analysis. Another approach to this problem is given recently by Newman (1993) who does a perturbation analysis where the low-frequency body oscillations are superposed on the diffraction field.

The effect of a uniform current on the first-order forces and a drift force component in the current direction has recently been studied by Grue & Palm (1985, 1986) in the two-dimensional case, and in the three-dimensional case by among others Zhao & Faltinsen (1989), Wu & Eatock-Taylor (1990) and Nossen, Grue & Palm (1991). It turns out that a current with speed $U = 1\ \mathrm{ms}^{-1}$, changes the magnitude of the drift forces acting on large-volume bodies significantly, of the order of 50% compared to the forces for $U = 0$. The effect of a current can therefore not be neglected. A current complicates the mathematics considerably and makes the linear velocities in the fluid more difficult to find.

There is, however, another effect of a current (or a forward speed) which has not, to our knowledge, been discussed in the literature. For $U = 0$, the drift forces and moments are completely determined by the linear solutions. This is not necessarily true when $U \neq 0$. We may then obtain contributions to the second-order forces and moments which are products of U and a steady second-order velocity. Assuming that the fluid motion is irrotational, the steady second-order velocity may be derived from a velocity potential, $\psi^{(2)}$. It is the intention in this paper to discuss these new terms. If U is moderate or large, it is complicated to evaluate $\psi^{(2)}$. This is essential because in the neighbourhood of the body, the boundary conditions for $\psi^{(2)}$ at the free surface become nonlinear. For many practical problems, however, U is small. Neglecting terms of $O(U^2)$, the mathematical problem of finding $\psi^{(2)}$ reduces to a linear boundary value problem. This is also the case when U is finite, but the body is slender (the Kelvin–Neumann problem).

There are two ways of obtaining the steady second-order forces and moments. The first one is by direct pressure integration over the body, the near-field method. This method has the merit that the local values of the forces and moments are obtained, which may lead to a better physical insight into the problem. This procedure leads to contributions which are proportional to the derivatives of $\psi^{(2)}$. It is, however, shown that for the horizontal drift forces and the yaw drift moment these contributions may be obtained without knowing the explicit solution for $\psi^{(2)}$, assuming that U is small. This is not true for the vertical drift force and the two other moments which require for their evaluation the complete solution of the boundary value problem.

The horizontal components of the drift force and the yaw moment may alternatively be obtained from, respectively, the linear and angular momentum fluxes in the far field. This way of proceeding has mathematical and numerical advantages, since the integrals are evaluated at a vertical control surface far away from the body. It turns also out that by this method the terms containing $\psi^{(2)}$ vanish in the expression for the horizontal forces. The physical reason for this is that in the linear momentum flux the contributions at infinity from the pressure term and the velocity term cancel each other. This is not true for the forces and the moment obtained by the near-field method, a result which is obvious since the momentum flux due to the velocity term is identically zero integrated over a rigid body.

For the yaw moment obtained by the far-field method we find, however, that $\psi^{(2)}$ makes a contribution. Indeed, $\psi^{(2)}$ may be as important as the terms from the products of the first-order quantities. The $\psi^{(2)}$-contribution is in the examples presented here, practically speaking, always found to increase the effect of the current on the moment. Interestingly, we find for realistic marine structures that the most important contribution from $\psi^{(2)}$ occurs for relatively long wavelengths where the wave spectrum very often has a maximum.

The $\psi^{(2)}$-field is found to be generated by the presence of first-order evanescent modes in the vicinity of the body and linear body motions. In the special case with restrained bodies having vertical boundaries extending deeply into the fluid, for example an array of vertical cylinders, the second-order velocities, $\nabla\psi^{(2)}$, are found to vanish exactly.

Formulae for the drift forces and moments are obtained in §2. Conservation of energy and mass are discussed in §3. In §4 we obtain far-field formulae for the drift force, yaw moment and energy flux for small values of the current speed, while §5 contains a detailed discussion of the $\psi^{(2)}$-contribution. Section 6 contains a discussion of results for finite values of the current speed while §7 is devoted to numerical results.

2. The steady second-order forces and moments

2.1. *The near field*

We consider a floating body moving horizontally with constant speed U and responding to long-crested incoming regular waves with small amplitude. Let us introduce a frame of reference (x, y, z) moving with forward speed U, in the same direction as the body, with the (x, y)-plane in the undisturbed free surface, the x-axis in the direction of the forward motion, and the z-axis positive upwards. Unit vectors $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ are introduced respectively along the (x, y, z)-directions. It is assumed that the motion is irrotational and the fluid incompressible. The total fluid velocity may then be written

$$\boldsymbol{v} = \nabla\Phi + U\nabla\chi_s. \tag{1}$$

Here χ_s is composed of the current potential, $-x$, and the steady velocity potential, χ, generated by the moving body, independent of the incoming waves, i.e.

$$\chi_s = -x+\chi; \tag{2}$$

Φ is the velocity potential due to incoming, scattered and radiated waves. χ and Φ both satisfy the Laplace equation. The corresponding fluid pressure is given by the Bernoulli equation

$$p = -\rho\left(\frac{\partial\Phi}{\partial t}+\tfrac{1}{2}U^2(|\nabla\chi_s|^2-1)+U\nabla\chi_s\cdot\nabla\Phi+\tfrac{1}{2}|\nabla\Phi|^2+gz\right)+C(t), \tag{3}$$

where $C(t)$ is an arbitrary function of time.

In this subsection we assume that the forces and the moments are obtained by pressure integration over the body (the near-field method). In the next subsection the same quantities will be examined using the far-field method. Φ may be written

$$\Phi = \phi^{(1)}+\phi^{(2)}+\psi^{(2)}, \tag{4}$$

where $\phi^{(1)}$ is the linear oscillatory potential proportional to the wave amplitude, and $\phi^{(2)}$ and $\psi^{(2)}$ are respectively the oscillatory and steady second-order potentials proportional to the wave amplitude squared. The terms $\frac{1}{2}U^2|\nabla\chi_s|^2$, gz, $\frac{1}{2}|\nabla\Phi|^2$, $\partial\Phi/\partial t$ and $U\nabla\chi_s\cdot\nabla\Phi$ in (3) give rise to steady second-order contributions, being products of two $\phi^{(1)}$-terms. The two latter terms also make contributions through products of $\phi^{(1)}$-terms and terms describing the first-order body motion. All these contributions are known when the complete first-order motions have been determined. The term $-\rho U\nabla\chi_s\cdot\nabla\Phi$, however, also gives rise to a term

$$-\rho U\int_{S_B}\nabla\chi_s\cdot\nabla\psi^{(2)}n_i\,\mathrm{d}S, \quad i = 1, 2, \ldots, 6, \tag{5}$$

where $(n_1, n_2, n_3) = \boldsymbol{n}$ and $(n_4, n_5, n_6) = (\boldsymbol{x}\times\boldsymbol{n})$, with $\boldsymbol{n}$ being the normal vector, positive out of the fluid and $\boldsymbol{x} = (x, y, z)$. S_B denotes integration over the wetted part of the body in the mean position. Also the term $\frac{1}{2}U^2|\nabla\chi_s|^2$ in (3) may give rise to a term similar to (5). We shall in this subsection neglect contributions of $O(U^2)$, however.

To evaluate (5) it seems necessary to compute the second-order velocity potential $\psi^{(2)}$. It is shown in §5 how this can be done for arbitrary bodies, provided U is small. We have to solve an integral equation and integrate over both the body and the free surface, which can be time consuming. However, the contributions from (5) to the horizontal force components and the yaw moment may, for small values of U be written in a proper form without solving for $\psi^{(2)}$. In §5 we show that the integrals (5) for these components may be replaced by integrals over the free surface and the body surface where $\psi^{(2)}$ enters in the form $\partial\psi^{(2)}/\partial n$. For the horizontal forces it is found that

$$-\rho U\int_{S_B} \nabla\chi_s\cdot\nabla\psi^{(2)} n_i\,\mathrm{d}S = -\rho U\int_{S_F+S_B} \frac{\partial\psi^{(2)}}{\partial n}\frac{\partial\chi}{\partial x_i}\mathrm{d}S,\quad i=1,2, \tag{6}$$

and for the yaw moment we obtain

$$-\rho U\int_{S_B} \nabla\chi_s\cdot\nabla\psi^{(2)} n_6\,\mathrm{d}S = -\rho U\int_{S_F+S_B} \Psi\frac{\partial\psi^{(2)}}{\partial n}\mathrm{d}S-\rho U\int_{S_F+S_B}\frac{\partial\psi^{(2)}}{\partial n}\left(x\frac{\partial\chi}{\partial y}-y\frac{\partial\chi}{\partial x}\right)\mathrm{d}S. \tag{7}$$

Here Ψ denotes the steady velocity potential if the body is moving along the positive y-axis (corresponding to χ when the body is moving along the positive x-axis). Ψ is the solution of the boundary value problem given in (63)–(66). S_F denotes integration over the free surface.

The integrals (6) and (7) may be further rewritten by expressing $\partial\psi^{(2)}/\partial n$ in terms of $\phi^{(1)}$ and the first-order body motions. Since U appears as a prefactor in (6) and (7) it suffices to determine $\partial\psi^{(2)}/\partial n$ for $U=0$. The free surface boundary condition for $\psi^{(2)}$ reads

$$\frac{\partial\psi^{(2)}}{\partial z} = -\frac{1}{g}\overline{\frac{\partial}{\partial t}\nabla\phi^{(1)}\cdot\nabla\phi^{(1)}}+\frac{1}{g^2}\overline{\frac{\partial\phi^{(1)}}{\partial t}\frac{\partial^3\phi^{(1)}}{\partial z\,\partial t^2}}+\frac{1}{g}\overline{\frac{\partial\phi^{(1)}}{\partial t}\frac{\partial^2\phi^{(1)}}{\partial z^2}}\quad\text{on}\quad z=0, \tag{8}$$

where a bar denotes time average. Introducing for the first-order potential

$$\phi^{(1)} = \mathrm{Re}(\phi e^{i\sigma t}), \tag{9}$$

where σ denotes the frequency of encounter (see (34)), and noting that the two first terms on the right-hand side of (8) vanish, we obtain

$$\frac{\partial\psi^{(2)}}{\partial z} = -\frac{\sigma}{2g}\mathrm{Im}\left(\phi\frac{\partial^2\phi^*}{\partial z^2}\right)\quad\text{on}\quad z=0. \tag{10}$$

Here an asterisk denotes complex conjugate.

Correspondingly, $\partial\psi^{(2)}/\partial n$ on the body boundary may be expressed in terms of the first-order potential and the first-order body motions. Applying the results derived by Ogilvie (1983) for the complete second-order potential, we find

$$\frac{\partial\psi^{(2)}}{\partial n} = V_n^{(2)}\quad\text{on}\quad S_B, \tag{11}$$

where $V_n^{(2)}$ is

$$V_n^{(2)} = -\overline{\boldsymbol{n}\cdot[(\boldsymbol{\xi}^{(1)}+\boldsymbol{\alpha}^{(1)}\times\boldsymbol{x})\cdot\nabla]\,\nabla\phi^{(1)}}+\overline{(\boldsymbol{\alpha}^{(1)}\times\boldsymbol{n})\cdot[\mathrm{d}/\mathrm{d}t(\boldsymbol{\xi}^{(1)}+\boldsymbol{\alpha}^{(1)}\times\boldsymbol{x})-\nabla\phi^{(1)}]}. \tag{12}$$

Here, $\boldsymbol{\xi}^{(1)} = \mathrm{Re}\{(\xi_1,\xi_2,\xi_3)\,e^{i\sigma t}\}$ and $\boldsymbol{\alpha}^{(1)} = \mathrm{Re}\{(\xi_4,\xi_5,\xi_6)\,e^{i\sigma t}\}$ denote respectively the first-order translations and rotations of the body.

Introducing (10)–(11) in (6)–(7) we see that $\psi^{(2)}$ in the integral (5) is replaced by products of first-order quantiites. The free-surface integrals on the right-hand sides in (6) and (7) are easily evaluated numerically since χ, Ψ, and $\partial\psi^{(2)}/\partial z$ decay rapidly away from the body.

For $i = 3, 4, 5$, we obtain, instead of (6) and (7) formulae where $\psi^{(2)}$ also enters. We have not been able to find $\psi^{(2)}$ expressed by products of first-order quantities (like (10) and (11), (12) for $\partial\psi^{(2)}/\partial n$). We therefore believe that for these components it is necessary to solve the integral equation for $\psi^{(2)}$ and then compute (5) directly.

2.2. *The far field*

An often used method to obtain the steady second-order forces and moments is to apply the principle of conservation of linear and angular momentum. Thereby the integration of the momentum may be replaced from the body surface to the restrained vertical cylinder at infinity. This procedure has the merit that the geometry is very simple and that the χ-field defined by (2) vanishes. Furthermore, the Green function and the velocity potentials for the wave motion may be replaced by their asymptotic values. These simplifications make it possible to use analytical methods in the evaluation of the forces and moments. Let S_∞ denote the surface of the vertical circular cylinder at infinity extending up to the free surface. The mean horizontal force may then be written

$$\boldsymbol{F} = -\overline{\int_{S_\infty} (p\boldsymbol{n} + \rho \boldsymbol{v} v_n)\, \mathrm{d}S}, \tag{13}$$

where a bar denotes the time average and $v_n = \boldsymbol{v}\cdot\boldsymbol{n}$. Correspondingly, using the principle of conservation of angular momentum, the yaw moment on the body is given by

$$M_z = -\boldsymbol{k}\cdot\overline{\int_{S_\infty} (p\boldsymbol{x}\times\boldsymbol{n} + \rho\boldsymbol{x}\times\boldsymbol{v} v_n)\, \mathrm{d}S}. \tag{14}$$

In the Bernoulli equation (3), $\overline{\Phi_t} = 0$, and for the second-order pressure $C(t) = O(A^2)$, where A is the amplitude of the incoming waves. Furthermore, the coupling term $\rho U \nabla\chi_s \cdot \nabla\psi^{(2)}$ reduces to $-\rho U\, \partial\psi^{(2)}/\partial x$.

Let us first consider the horizontal forces given by (13). We notice that the last term in (13) gives rise to two terms containing U and $\psi^{(2)}$, namely

$$-\rho U \boldsymbol{i}\overline{\int_{S_\infty} v_n\, \mathrm{d}S} - \rho U \int_{S_\infty} n_1 \nabla\psi^{(2)}\, \mathrm{d}S. \tag{15}$$

Here the first term is zero due to conservation of mass, and the x-component of the last term cancels the contribution from the pressure term in (13). Hence, by the far-field method this component of the steady second-order force is completely determined by the first-order quantities, for all values of U. In the y-direction the coupling term gives rise to a force

$$\rho U \int_{S_\infty} \left(n_1 \frac{\partial\psi^{(2)}}{\partial y} - n_2 \frac{\partial\psi^{(2)}}{\partial x} \right) \mathrm{d}S = \rho U \int_{S_\infty} \frac{\partial\psi^{(2)}}{\partial s}\, \mathrm{d}s\, \mathrm{d}z, \tag{16}$$

where s here denotes the arclength of a circle with a constant z-value.

To obtain the explicit form of the horizontal steady second-order force, we introduce (3) in (13) and find

$$\boldsymbol{F} = -\overline{\int_{S_\infty} \left(-\rho(\Phi_t + \tfrac{1}{2}|\nabla\phi^{(1)}|^2 + gz)\boldsymbol{n} + \rho\nabla\phi^{(1)} \frac{\partial\phi^{(1)}}{\partial n} \right) \mathrm{d}S} - \overline{C(t) \int_{S_\infty} \boldsymbol{n}\, \mathrm{d}S}$$
$$+ \rho U \boldsymbol{i} \overline{\int_{S_\infty} v_n\, \mathrm{d}S} + \rho U \overline{\int_{S_\infty} \left(n_1 \nabla\Phi - \boldsymbol{n}\frac{\partial\Phi}{\partial x} \right) \mathrm{d}S}. \tag{17}$$

The third term on the right-hand side of (17) vanishes since the mass is conserved. After some algebra $\boldsymbol{F}$ may be written

$$\boldsymbol{F}=\rho\int_{C_\infty}\overline{\frac{1}{2g}\left(\left(\frac{\partial\phi^{(1)}}{\partial t}\right)^2-U^2\left(\frac{\partial\phi^{(1)}}{\partial x}\right)^2\right)\boldsymbol{n}}\,\mathrm{d}s+\rho\int_{S_\infty}\left(\overline{\tfrac{1}{2}(\nabla\phi^{(1)})^2\boldsymbol{n}-\nabla\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial n}}\right)\mathrm{d}S$$
$$+\rho U\boldsymbol{j}\left(\int_{C_\infty}\overline{\zeta^{(1)}\frac{\partial\phi^{(1)}}{\partial s}}\,\mathrm{d}s+\int_{S_\infty}\frac{\partial\psi^{(2)}}{\partial s}\,\mathrm{d}s\,\mathrm{d}z\right),\quad (18).$$

where C_∞ denotes the waterline of S_∞ and $\zeta^{(1)}$ the first-order free-surface elevation. Formula (18) is consistent with Maruo (1960), Zhao & Faltinsen (1990) and Kashiwagi (1991), except for the last term due to the second-order velocity, which is absent in their derivation. Obviously this term is zero if the velocity circulation in the fluid is zero. In some special cases, however, as for example a boat sailing at a non-zero angle of attack, it is expected that a steady velocity circulation is present, giving rise to the lifting force in (18). Then the effect of a possible trailing vortex sheet must be accounted for.

For the steady second-order yaw moment we obtain, using that the contribution from the pressure field vanishes,

$$M_z=-\rho\int_{S_\infty}\overline{\frac{\partial\phi^{(1)}}{\partial\theta}\frac{\partial\phi^{(1)}}{\partial R}}\,\mathrm{d}S-\rho U\int_{C_\infty}\overline{\left(y\frac{\partial\phi^{(1)}}{\partial R}-\frac{x}{R}\frac{\partial\phi^{(1)}}{\partial\theta}\right)\zeta^{(1)}}\,\mathrm{d}s$$
$$-\rho U\int_{S_\infty}\left(y\frac{\partial\psi^{(2)}}{\partial R}-\frac{x}{R}\frac{\partial\psi^{(2)}}{\partial\theta}\right)\mathrm{d}S+\rho U^2\int_{C_\infty}yn_1\overline{\zeta^{(2)}}\,\mathrm{d}s,\quad (19)$$

where R is the radius of the cylinder, θ the polar angle and $\zeta^{(2)}$ is the second-order free-surface elevation. The formula (19) is valid for arbitrary U and water depth. We notice that the two first terms on the right-hand side are products of first-order quantities. The last term will be neglected, being of $O(U^2)$. As shown in §5 the third term may for small values of U be recast into the form

$$-\rho U\int_{S_\infty}\left(y\frac{\partial\psi^{(2)}}{\partial R}-\frac{x}{R}\frac{\partial\psi^{(2)}}{\partial\theta}\right)\mathrm{d}S=-\rho U\left(\int_{S_\mathrm{B}}\psi^{(2)}n_2\,\mathrm{d}S-\int_{S_\mathrm{F}+S_\mathrm{B}}y\frac{\partial\psi^{(2)}}{\partial n}\mathrm{d}S\right)$$
$$=-\rho U\int_{S_\mathrm{F}+S_\mathrm{B}}\varPsi_\mathrm{s}\frac{\partial\psi^{(2)}}{\partial n}\mathrm{d}S.\quad (20)$$

Here $\varPsi_\mathrm{s}$ is the steady velocity potential if the body is moving along the positive y-axis (corresponding to χ_s when the body is moving along the positive x-axis). Analogous to (2) we write

$$\varPsi_\mathrm{s}=\varPsi-y.\quad (21)$$

Using (10) and (11), M_z is given by

$$M_z=-\rho\int_{S_\infty}\overline{\frac{\partial\phi^{(1)}}{\partial\theta}\frac{\partial\phi^{(1)}}{\partial R}}\,\mathrm{d}S-\rho U\int_{C_\infty}\overline{\left(y\frac{\partial\phi^{(1)}}{\partial R}-\frac{x}{R}\frac{\partial\phi^{(1)}}{\partial\theta}\right)\zeta^{(1)}}\,\mathrm{d}s$$
$$+\frac{\rho U\sigma}{2g}\int_{S_\mathrm{F}}\varPsi_\mathrm{s}\,\mathrm{Im}\left(\phi\frac{\partial^2\phi^*}{\partial z^2}\right)\mathrm{d}S-\rho U\int_{S_\mathrm{B}}\varPsi_\mathrm{s}V_n^{(2)}\,\mathrm{d}S,\quad (22)$$

where $V_n^{(2)}$ is given by (12). Thus, M_z is expressed entirely in terms of first-order quantities.

For $U=0$, (22) becomes identical to the expression derived by Newman (1967) for zero forward speed. A formula for M_z with non-zero forward velocity has recently been derived by Kashiwagi (1991). However, he does not consider the effect of the steady second-order velocity.

3. Conservation of energy and mass

The energy flux at infinity is given by

$$W = \overline{\int_{S_\infty} (p + \tfrac{1}{2}\rho v^2 + \rho g z)\, \boldsymbol{v}\cdot\boldsymbol{n}\, \mathrm{d}S}. \tag{23}$$

Inserting for the pressure we obtain, to second order,

$$W = -\rho \int_{S_\infty} \overline{\frac{\partial \phi^{(1)}}{\partial t}\frac{\partial \phi^{(1)}}{\partial n}}\, \mathrm{d}S + \rho U \int_{C_\infty} \overline{\frac{\partial \phi^{(1)}}{\partial t}\zeta^{(1)}} n_1\, \mathrm{d}s. \tag{24}$$

We notice that the energy flux at S_∞ is completely determined by first-order quantities, a result which was obtained earlier for the two-dimensional case by Grue & Palm (1985).

The mean flux of mass through the infinite cylinder is given by

$$\rho \overline{\int_{S_\infty} v_n\, \mathrm{d}S} = \rho \int_{C_\infty} \overline{\frac{\partial \phi^{(1)}}{\partial n}\zeta^{(1)}}\, \mathrm{d}s - \rho U \int_{C_\infty} \overline{\zeta^{(2)}} n_1\, \mathrm{d}s + \rho \int_{S_\infty} \frac{\partial \psi^{(2)}}{\partial n}\, \mathrm{d}S. \tag{25}$$

Since the motion is periodic in time, the mean mass transport through the cylinder, averaged over time, is zero. The first term on the right in (25) is the mass transport due to the Stokes' drift. This is balanced by the mean second-order velocity and a term due to the second-order deflection of the free surface at infinity.

Also, the integrated normal velocity of the free surface, averaged over time, is zero. Hence, to leading order

$$\rho \overline{\int_{S_F} v_n\, \mathrm{d}S} = \rho U \int_{S_F} \frac{\partial \chi_s}{\partial z}\mathrm{d}S + \rho \int_{S_F} \frac{\partial \psi^{(2)}}{\partial z}\mathrm{d}S + \rho \int_{S_F} \left(\overline{\boldsymbol{n}_H\cdot\nabla\phi^{(1)}} + \overline{\frac{\partial^2 \phi^{(1)}}{\partial z^2}\zeta^{(1)}} \right) \mathrm{d}S = 0, \tag{26}$$

where $\boldsymbol{n}_H = (n_1, n_2)$. Since both $\partial\chi/\partial n$ and $\partial\psi^{(2)}/\partial n$ integrated over $S_F + S_\infty$ are zero, it follows from (25) and (26) that the last integral in (26) is equal to the Stokes' drift at infinity, with opposite sign. This may also be seen by direct evaluation of the integral.

Let us assume for the moment that $U = 0$. From (24) we then obtain

$$W = -\rho \int_{S_\infty} \overline{\frac{\partial \phi^{(1)}}{\partial t}\frac{\partial \phi^{(1)}}{\partial n}}\, \mathrm{d}S = -\frac{\rho c c_g}{g} \int_{C_\infty} \overline{\frac{\partial \phi^{(1)}}{\partial t}\frac{\partial \phi^{(1)}}{\partial n}}\, \mathrm{d}s = \rho c c_g D, \tag{27}$$

where c and c_g are the phase velocity and group velocity, respectively, and D is the Stokes' drift (at infinity)

$$D = \int_{C_\infty} \overline{\zeta^{(1)}\frac{\partial \phi^{(1)}}{\partial n}}\, \mathrm{d}s. \tag{28}$$

From (25) it follows that for $U = 0$, D is also equal to

$$D = -\int_{S_\infty} \frac{\partial \psi^{(2)}}{\partial n}\, \mathrm{d}S. \tag{29}$$

If the body is performing no work on the fluid over a period, $W = 0$. This is true for a body drifting freely in the waves or a body restrained from moving. It then follows from (27), (28), and (29) that the Stokes' drift (at infinity) and the mass flux due to the $\psi^{(2)}$-field over the free surface are zero. The latter result will be used in §5.

4. Formulae for the drift force, mean yaw moment and energy flux for small values of U

The horizontal drift forces, obtained by the far-field analysis, are given by (18). For simplicity we assume that the velocity circulation is zero. Furthermore, using (35) and (36) below, it may be shown that

$$\rho U \int_{C_\infty} \overline{\zeta^{(1)} \frac{\partial \phi^{(1)}}{\partial s}} \, \mathrm{d}s = O(U^2). \tag{30}$$

Hence, for small values of U, we have

$$\boldsymbol{F} = \rho \int_{C_\infty} \frac{1}{2g} \overline{\left(\frac{\partial \phi^{(1)}}{\partial t}\right)^2} \boldsymbol{n} \, \mathrm{d}s + \rho \int_{S_\infty} \overline{\left(\tfrac{1}{2}(\nabla \phi^{(1)})^2 \boldsymbol{n} - \nabla \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial n}\right)} \mathrm{d}S + O(U^2). \tag{31}$$

For the mean yaw moment we have to leading order in U

$$M_z = -\rho \int_{S_\infty} \overline{\frac{\partial \phi^{(1)}}{\partial \theta} \frac{\partial \phi^{(1)}}{\partial R}} \mathrm{d}S - \rho U \int_{C_\infty} \overline{\left(y \frac{\partial \phi^{(1)}}{\partial R} - \frac{x}{R} \frac{\partial \phi^{(1)}}{\partial \theta}\right) \zeta^{(1)}} \, \mathrm{d}s$$
$$-\rho U \int_{S_\infty} \left(y \frac{\partial \psi^{(2)}}{\partial R} - \frac{x}{R} \frac{\partial \psi^{(2)}}{\partial \theta}\right) \mathrm{d}S + O(U^2). \tag{32}$$

Let us introduce for the first-order potential defined in (9), with ϕ being decomposed as

$$\phi = A \frac{\mathrm{i}g}{\omega} (\phi_0 + \phi_\mathrm{B}), \tag{33}$$

where A denotes the amplitude and ω the frequency of the incoming wave (for $U = 0$); ω is related to the wavenumber K by $\omega^2 = gK$. The encounter frequency σ and ω are related by

$$\sigma = \omega - UK \cos \beta \tag{34}$$

The incoming wave potential ϕ_0 is

$$\phi_0 = \exp\left[Kz - \mathrm{i}K(x \cos \beta + y \sin \beta)\right], \tag{35}$$

where β denotes the incidence angle. ϕ_B represents the sum of the diffraction and radiation potentials and in the far field is given by

$$\phi_\mathrm{B} = R^{-\frac{1}{2}} H(\theta) \exp\left[k_1(\theta)\left(z - \mathrm{i}R(1 - 4\tau^2 \sin^2 \theta)^{\frac{1}{2}}\right)\right] + O(1/R) \quad \text{as} \quad R \to \infty, \tag{36}$$

where

$$\tau = U\sigma/g, \tag{37}$$

$$k_1 = \nu(1 + 2\tau \cos \theta) + O(\tau^2), \tag{38}$$

$$\nu = \sigma^2/g = K(1 - 2\tau \cos \beta) + O(\tau^2), \tag{39}$$

and $H(\theta)$ is the amplitude distribution of the potential. The formula for this is given in Nossen *et al.* (1991, equation 64, 70, 71). Introducing (9), (33), (35), (36), averaging with respect to time, and applying the method of stationary phase, we obtain for F_x and F_y

$$\frac{F_x}{\rho g A^2} = -\frac{\nu}{4K}\left\{\int_0^{2\pi} (\cos\theta + 2\tau\sin^2\theta)|H(\theta)|^2\,\mathrm{d}\theta + 2\cos\beta\,\mathrm{Re}\,(S)\right\} + O(\tau^2), \quad (40)$$

$$\frac{F_y}{\rho g A^2} = -\frac{\nu}{4K}\left\{\int_0^{2\pi} (\sin\theta - 2\tau\sin\theta\cos\theta)|H(\theta)|^2\,\mathrm{d}\theta + 2\sin\beta\mathrm{Re}(S)\right\} + O(\tau^2), \quad (41)$$

where
$$S = (2\pi/\nu)^{\frac{1}{2}}\mathrm{e}^{\mathrm{i}\pi/4}H^*(\beta + 2\tau\sin\beta). \quad (42)$$

As will be shown in §5 and the Appendix we obtain for the yaw moment

$$\frac{M_z}{\rho g A^2} = -\frac{1}{4K}\mathrm{Im}\left\{\int_0^{2\pi}(1-2\tau\cos\theta)\,H\frac{\mathrm{d}H^*}{\mathrm{d}\theta}\,\mathrm{d}\theta\right\} - \frac{1}{2K}\mathrm{Im}\left\{\frac{\nu}{K}S' + \tau\sin\beta S\right\}$$
$$+\frac{\tau}{2K}\int_{S_F}\Psi_s\,\mathrm{Im}\left[(\phi_0+\phi_B)\frac{\partial^2}{\partial z^2}(\phi_0^*+\phi_B^*)\right]\mathrm{d}x\,\mathrm{d}y - \frac{\tau}{\sigma A^2}\int_{S_B}\Psi_s\,V_n^{(2)}\,\mathrm{d}S + O(\tau^2), \quad (43)$$

where $V_n^{(2)}$ is given by (12), S by (42) and S' by

$$S' = (2\pi/\nu)^{\frac{1}{2}}\mathrm{e}^{\mathrm{i}\pi/4}(H'(\beta+2\tau\sin\beta))^*. \quad (44)$$

Here H' denotes a derivative with respect to the argument.

For the energy flux we obtain from (24)

$$\frac{W}{Ec_g} = -\frac{\sigma}{2\omega}\left\{\int_0^{2\pi}(1-2\tau\cos\theta)|H(\theta)|^2\,\mathrm{d}\theta + 2\frac{\nu}{K}\mathrm{Re}(S)\right\} + O(\tau^2), \quad (45)$$

where $E \equiv \frac{1}{2}\rho g A^2$ and c_g denote the mean energy density and the group velocity of the incoming wave, respectively.

5. The steady second-order velocity potential

5.1. *General*

The steady second-order velocity enters in the equations only multiplied with U. To first order in U we therefore only need to consider $\psi^{(2)}$ for $U = 0$. $\psi^{(2)}$ is determined by

$$\nabla^2\psi^{(2)} = 0, \quad (46)$$

$$\frac{\partial\psi^{(2)}}{\partial z} = -\frac{\sigma}{2g}\mathrm{Im}\left(\phi\frac{\partial^2\phi^*}{\partial z^2}\right) \quad \text{on} \quad z = 0, \quad (47)$$

$$\partial\psi^{(2)}/\partial n = V_n^{(2)} \quad \text{on} \quad S_B, \quad (48)$$

$$\nabla\psi^{(2)} \to 0, \quad |\boldsymbol{x}| \to \infty, \quad (49)$$

where we have used (10), and $V_n^{(2)}$ is given by (12).

In the general case we note that the free-surface boundary condition $\mathrm{Im}\,(\phi\partial^2\phi^*/\partial z^2)$ vanishes far away from the body since there

$$\phi(x,y,z) = \mathrm{e}^{Kz}\phi(x,y,0), \quad (50)$$

where K is the wavenumber. This conclusion is, however, not true close to the body, except in the special case when the body is restrained and has vertical walls extending deeply in the fluid. Then, $\partial\psi^{(2)}/\partial z = 0$ at S_F and $\partial\psi^{(2)}/\partial n = 0$ at S_B. In this case the steady second-order velocity is zero in the entire fluid region. An immediate conclusion is then that the $\psi^{(2)}$-field is generated by the presence of first-order evanescent modes in the vicinity of the body and due to linear body motions.

$\psi^{(2)}$ is obtained from (46)–(49) by using Green's formula on $\psi^{(2)}$ and $G = 1/r + 1/r_1$ where $r = |(x,y,z)-(x',y',z')|$ and $r_1 = |(x,y,z)-(x',y',-z')|$. Here (x,y,z) and (x',y',z') denote the space and source coordinates, respectively. Utilizing that the integral of $(\psi^{(2)}\,\partial G/\partial n - (\partial\psi^{(2)}/\partial n)\,G)$ over S_∞ vanishes and that $\partial G/\partial n = 0$ on S_F, we obtain

$$4\pi\psi^{(2)}(\boldsymbol{x}) = -\int_{S_B} \psi^{(2)}\frac{\partial G}{\partial n}\,\mathrm{d}S + \int_{S_F+S_B} \frac{\partial\psi^{(2)}}{\partial n}G\,\mathrm{d}S, \tag{51}$$

$$2\pi\psi^{(2)}(\boldsymbol{x}) = -\int_{S_B} \psi^{(2)}\frac{\partial G}{\partial n}\,\mathrm{d}S + \int_{S_F+S_B} \frac{\partial\psi^{(2)}}{\partial n}G\,\mathrm{d}S, \tag{52}$$

where in (51) $\boldsymbol{x}$ is in the fluid domain and in (52) $\boldsymbol{x}$ is an element of S_B. The latter is an integral equation for $\psi^{(2)}$, since $\partial\psi^{(2)}/\partial n$ is known over S_F+S_B. It is, however, not necessary to solve the integral equation to eliminate $\psi^{(2)}$ in the formulae for the horizontal forces and yaw moment, as pointed out in §2. Using the near-field method, it is sufficient to prove that formulae (6)–(7) hold. In the far-field method, the expressions for the forces do not contain $\psi^{(2)}$, whereas $\psi^{(2)}$ is eliminated in the formula for the yaw moment if (20) is valid. To prove (6)–(7) we use a variant of Stokes' theorem, valid for bodies that are wall-sided at the waterline. From this theorem it follows that

$$\int_{S_B} \nabla\chi_s\cdot\nabla\psi^{(2)}n_i\,\mathrm{d}S = -\int_{S_B}\psi^{(2)}m_i\,\mathrm{d}S - \int_{C_B}\psi^{(2)}\frac{\partial\chi_s}{\partial z}n_i\,\mathrm{d}s, \tag{53}$$

where $(m_1,m_2,m_3) = -\boldsymbol{n}\cdot\nabla\nabla\chi_s$, $(m_4,m_5,m_6) = -\boldsymbol{n}\cdot\nabla(\boldsymbol{x}\times\nabla\chi_s)$ and C_B denotes the waterline of the body. We first note that for small values of U the free-surface condition for the steady potential can be approximated by

$$\partial\chi_s/\partial z = 0 \quad \text{at} \quad z = 0. \tag{54}$$

Hence the last term in (53) may be neglected.

To transform the first integral on the right-hand side of (53), we introduce $\boldsymbol{\Theta} = \nabla\chi$ or $\boldsymbol{\Theta} = \boldsymbol{x}\times\nabla\chi$. Since $\boldsymbol{\Theta}$ satisfies the Laplace equation, Green's theorem gives

$$\int_{S_B+S_F+S_\infty}\left(\psi^{(2)}\frac{\partial\boldsymbol{\Theta}}{\partial n} - \frac{\partial\psi^{(2)}}{\partial n}\boldsymbol{\Theta}\right)\mathrm{d}S = 0. \tag{55}$$

Since the integral over S_∞ vanishes we obtain

$$\int_{S_B}\psi^{(2)}\frac{\partial\boldsymbol{\Theta}}{\partial n}\,\mathrm{d}S = \int_{S_F+S_B}\frac{\partial\psi^{(2)}}{\partial n}\boldsymbol{\Theta}\,\mathrm{d}S - \int_{S_F}\psi^{(2)}\frac{\partial\boldsymbol{\Theta}}{\partial z}\,\mathrm{d}S. \tag{56}$$

Introducing (56) in (53) and applying that $\partial\chi/\partial z = 0$ at the free surface, we obtain for the horizontal force components

$$-\rho U\int_{S_B}\nabla\chi_s\cdot\nabla\psi^{(2)}n_i\,\mathrm{d}S = -\rho U\int_{S_F+S_B}\frac{\partial\psi^{(2)}}{\partial n}\frac{\partial\chi}{\partial x_i}\,\mathrm{d}S, \quad i = 1,2. \tag{57}$$

Thus we have demonstrated (6). For the vertical force we obtain

$$-\rho U\int_{S_B}\nabla\chi_s\cdot\nabla\psi^{(2)}n_3\,\mathrm{d}S = -\rho U\int_{S_B}\frac{\partial\psi^{(2)}}{\partial n}\frac{\partial\chi}{\partial z}\mathrm{d}S+\rho U\int_{S_F}\psi^{(2)}\frac{\partial^2\chi}{\partial z^2}\mathrm{d}S, \tag{58}$$

which involves evaluation of $\psi^{(2)}$ over the free surface.

For the moment we have to add the term

$$-\rho U\int_{S_B}\psi^{(2)}\boldsymbol{n}\cdot\nabla(\boldsymbol{x}\times-\boldsymbol{i})\,\mathrm{d}S = -\rho U\int_{S_B}\psi^{(2)}(n_2\boldsymbol{k}-n_3\boldsymbol{j})\,\mathrm{d}S. \tag{59}$$

We then obtain for the yaw moment

$$-\rho U\int_{S_B}\nabla\chi_s\cdot\nabla\psi^{(2)}n_6\,\mathrm{d}S = -\rho U\int_{S_B}\psi^{(2)}n_2\,\mathrm{d}S-\rho U\int_{S_F+S_B}\frac{\partial\psi^{(2)}}{\partial n}\left(x\frac{\partial\chi}{\partial y}-y\frac{\partial\chi}{\partial x}\right)\mathrm{d}S, \tag{60}$$

and for the horizontal moments

$$-\rho U\int_{S_B}\nabla\chi_s\cdot\nabla\psi^{(2)}n_4\,\mathrm{d}S = \rho U\int_{S_F}\psi^{(2)}\left(y\frac{\partial^2\chi}{\partial z^2}-\frac{\partial\chi}{\partial y}\right)\mathrm{d}S-\rho U\int_{S_B}\frac{\partial\psi^{(2)}}{\partial n}\left(y\frac{\partial\chi}{\partial z}-z\frac{\partial\chi}{\partial y}\right)\mathrm{d}S, \tag{61}$$

$$-\rho U\int_{S_B}\nabla\chi_s\cdot\nabla\psi^{(2)}n_5\,\mathrm{d}S = \rho U\int_{S_B}\psi^{(2)}n_3\,\mathrm{d}S+\rho U\int_{S_F}\psi^{(2)}\left(\frac{\partial\chi}{\partial x}-x\frac{\partial^2\chi}{\partial z^2}\right)\mathrm{d}S$$
$$-\rho U\int_{S_B}\frac{\partial\psi^{(2)}}{\partial n}\left(z\frac{\partial\chi}{\partial x}-x\frac{\partial\chi}{\partial z}\right)\mathrm{d}S. \tag{62}$$

To transfer the integral $\int_{S_B}\psi^{(2)}n_2\,\mathrm{d}S$ in (60) to an integral involving $\partial\psi^{(2)}/\partial n$, we introduce the function Ψ defined by

$$\nabla^2\Psi = 0, \tag{63}$$

$$\partial\Psi/\partial n = n_2 \quad \text{on} \quad S_B, \tag{64}$$

$$\partial\Psi/\partial z = 0 \quad \text{on} \quad S_F, \tag{65}$$

$$\nabla\Psi \to 0, \quad |\boldsymbol{x}|\to\infty. \tag{66}$$

Applying Green's theorem to $\psi^{(2)}$ and Ψ we obtain

$$\int_{S_B+S_F}\left(\psi^{(2)}\frac{\partial\Psi}{\partial n}-\frac{\partial\psi^{(2)}}{\partial n}\Psi\right)\mathrm{d}S = 0 \tag{67}$$

since the contribution from S_∞ vanishes. Using that $\partial\Psi/\partial n = 0$ at S_F, we obtain

$$\int_{S_B}\psi^{(2)}n_2\,\mathrm{d}S = \int_{S_F+S_B}\Psi\frac{\partial\psi^{(2)}}{\partial n}\mathrm{d}S. \tag{68}$$

Introducing (68) in (60), the latter becomes identical to (7). We note that the formulae derived are very useful for the horizontal forces and yaw moment, but of no use for the vertical force and the horizontal moments since we do not know $\psi^{(2)}$ explicitly for $z = 0$.

5.2. *The far-field behaviour of* $\psi^{(2)}$

To derive the useful formula (20) for the yaw moment by the far-field method we consider the identity

$$\int_V \nabla\cdot y\nabla\psi^{(2)}\,\mathrm{d}V = \boldsymbol{j}\cdot\int_V \nabla\psi^{(2)}\,\mathrm{d}V, \tag{69}$$

where V denotes the fluid domain. By using Gauss' theorem for both integrals we obtain

$$\int_{S_\infty+S_F+S_B} y\frac{\partial\psi^{(2)}}{\partial n}\,\mathrm{d}S = \int_{S_\infty+S_F+S_B} \psi^{(2)} n_2\,\mathrm{d}S. \tag{70}$$

Since $n_2 = O(A)$ on S_F, the equation reduces to

$$\int_{S_\infty+S_F+S_B} y\frac{\partial\psi^{(2)}}{\partial n}\,\mathrm{d}S = \int_{S_\infty+S_B} \psi^{(2)} n_2\,\mathrm{d}S. \tag{71}$$

By partial integration, assuming that the velocity circulation is zero,

$$\int_{S_\infty} \psi^{(2)} n_2\,\mathrm{d}S = \int_{S_\infty} \frac{x}{R}\frac{\partial\psi^{(2)}}{\partial\theta}\,\mathrm{d}S. \tag{72}$$

Hence, using (68)

$$-\rho U\int_{S_\infty}\left(y\frac{\partial\psi^{(2)}}{\partial R} - \frac{x}{R}\frac{\partial\psi^{(2)}}{\partial\theta}\right)\mathrm{d}S = -\rho U\int_{S_F+S_B} \Psi_s\frac{\partial\psi^{(2)}}{\partial n}\,\mathrm{d}S, \tag{73}$$

where

$$\Psi_s = \Psi - y \tag{74}$$

which is formula (20).

This result may also be obtained from (51), without solving the integral equation (52). For large values of $\boldsymbol{x}$, G and $\partial G/\partial n$ become

$$G = \frac{2}{|\boldsymbol{x}|} - 2\boldsymbol{x}'_H\cdot\nabla\frac{1}{|\boldsymbol{x}|} + \dots, \tag{75}$$

$$\frac{\partial G}{\partial n} = -2\boldsymbol{n}_H\cdot\nabla\frac{1}{|\boldsymbol{x}|} + \dots, \tag{76}$$

where $\boldsymbol{x}'_H = (x', y', 0)$ and $\boldsymbol{n}_H = (n_1, n_2, 0)$. Inserting (75) and (76) into (51) we have for $\psi^{(2)}$ far away from the body

$$\psi^{(2)} = \frac{Q}{2\pi|\boldsymbol{x}|} + \boldsymbol{M}\cdot\nabla\frac{1}{2\pi|\boldsymbol{x}|} + \dots, \tag{77}$$

where

$$Q = \int_{S_F+S_B} \frac{\partial\psi^{(2)}}{\partial n}\,\mathrm{d}S = -\int_{S_\infty} \frac{\partial\psi^{(2)}}{\partial n}\,\mathrm{d}S \tag{78}$$

and

$$\boldsymbol{M} = (M_1, M_2) = \int_{S_B} \psi^{(2)}\boldsymbol{n}_H\,\mathrm{d}S - \int_{S_F+S_B} \frac{\partial\psi^{(2)}}{\partial n}\boldsymbol{x}'_H\,\mathrm{d}S. \tag{79}$$

Thus

$$-\rho U\int_{S_\infty}\left(y\frac{\partial\psi^{(2)}}{\partial R} - \frac{x}{R}\frac{\partial\psi^{(2)}}{\partial\theta}\right)\mathrm{d}S = -\rho U M_2. \tag{80}$$

Using (68), $-\rho U M_2$ becomes equal to the right-hand side of (73), and the formulae (73) and (80) become identical. We note that the first term of on the right of (77) is a sink with strength Q which equals the Stokes' drift, see (28) and (29), and is proportional to the net work performed by the body on the fluid, see (27). When the body is performing no work on the fluid we have that $Q = 0$.

6. Comments on the second-order velocity for finite values of U

The problem of finding $\psi^{(2)}$ for arbitrary values of U is important, but complicated. This is essentially because the total χ_s-field is of order unity and the corresponding elevation of the free surface is finite. The problem is somewhat simplified if the body is assumed to be slender, but it is still necessary to make additional simplifications. One possibility is to use the Dawson approximation by which it is assumed that $\partial\chi_s/\partial z = 0$ at $z = 0$ for finite values of U also (see e.g. Nakos & Sclavounos 1990). Another possibility is to replace the problem by the Kelvin–Neumann problem where in χ_s the χ-field is totally neglected so that χ_s is replaced by $-x$. Both of these approximations contain inconsistencies. Using the latter, which is the simplest one, the boundary condition for $\psi^{(2)}$ is by this approximation given by

$$\frac{U^2}{g}\frac{\partial^2\psi^{(2)}}{\partial x^2}+\frac{\partial\psi^{(2)}}{\partial z}=f(x,y) \quad \text{on} \quad z=0, \tag{81}$$

where $$f(x,y) = -\frac{1}{g}\frac{\partial}{\partial t'}\overline{\nabla\phi^{(1)}\cdot\nabla\phi^{(1)}}+\frac{1}{g^2}\overline{\frac{\partial\phi^{(1)}}{\partial t'}\frac{\partial^3\phi^{(1)}}{\partial z\,\partial t'^2}}+\frac{1}{g}\overline{\frac{\partial\phi^{(1)}}{\partial t'}\frac{\partial^2\phi^{(1)}}{\partial z^2}} \quad \text{on} \quad z=0. \tag{82}$$

Here $f(x,y)$ is identical to the right-hand side of (8) when $\partial/\partial t$ is replaced by $\partial/\partial t'$ where

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial t}-U\frac{\partial}{\partial x}. \tag{83}$$

In addition, $\psi^{(2)}$ satisfies the Laplace equation, a body boundary condition and a far-field condition.

We shall not discuss here the solution of this boundary value problem, which may be obtained by using integral equation technique. We note, however, that (20) for the yaw moment computed by the far-field method is valid for all values of U. Furthermore, we note that the body boundary condition for $\psi^{(2)}$ contains only products between (known) linear quantities. Hence, introducing in (20) $\partial\psi^{(2)}/\partial z$ given by (81) and (82), the term $U^2\,\partial^2\psi^{(2)}/\partial x^2$ gives rise to a third-order term in U. Thus, by neglecting this term we have a solution for the yaw moment, expressed by known first-order quantities, valid to $O(U^2)$.

7. Evaluation of the $\psi^{(2)}$-contribution for small values of U

7.1. *The far-field method; the diffraction problem*

Attention is primarily focused here on the contribution due to $\psi^{(2)}$. Since $\psi^{(2)}$ does not contribute to the horizontal forces in the far-field method, only the mean yaw moment is considered. According to (19)–(21) the contribution due to $\psi^{(2)}$ is given by

$$\rho U\int_{S_F+S_B} y\frac{\partial\psi^{(2)}}{\partial n}\,\mathrm{d}S-\rho U\int_{S_F+S_B}\Psi\frac{\partial\psi^{(2)}}{\partial n}\,\mathrm{d}S. \tag{84}$$

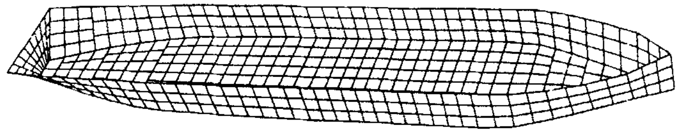

FIGURE 1. The ship model.

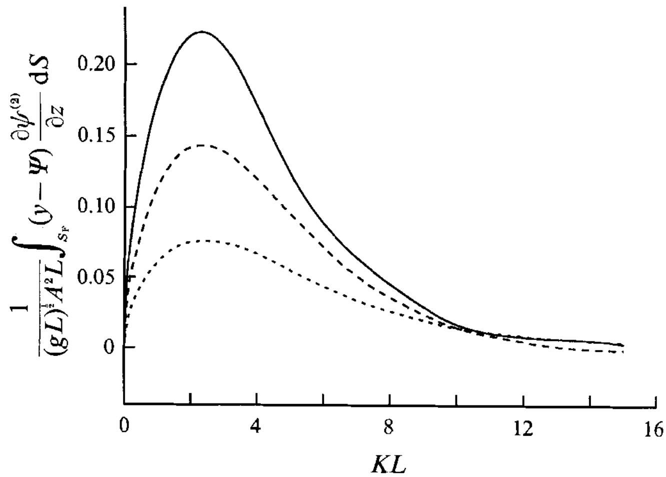

FIGURE 2. Non-dimensional values of $\int_{S_F}(y-\Psi)\,\partial\psi^{(2)}/\partial z\,\mathrm{d}S$ for the restrained ship ($L = 230$ m) *vs.* non-dimensional wavenumber KL: solid line, $\beta = 90°$; dashed line, $\beta = 140°$; dotted line, $\beta = 160°$.

It is assumed in this subsection that the marine structure is restrained from moving. According to the boundary condition (48) the integration over S_B in (84) then vanishes.

As a numerical example we consider a ship with length $L = 230$ m and beam $B = 41$ m, see figure 1. The angle of incidence for the waves, defined as the angle between the forward speed direction and the wave direction, is 90°, 140° and 160° (180° corresponds to head waves).

For small forward speed the mean yaw moment may be expressed in the form

$$M_z = M_{z0} + FrM_{z1} + O(Fr^2), \tag{85}$$

where M_{z0} denotes the moment for $U = 0$, $Fr = U/(gL)^{\frac{1}{2}}$ denotes the Froude number and FrM_{z1} gives the change in the moment due to U. In figure 2 (84) is shown as a function of the non-dimensional wavenumber KL. We notice that the function has its maximum value for very long waves and for angle of incidence equal to 90°. In figure 3(*a–c*) the numerical results for the total value of M_{z1} and for the value without the $\psi^{(2)}$-contribution are displayed as a function of KL. We notice that for shorter waves, KL larger than 10, say, the contribution from $\psi^{(2)}$ vanishes whereas for very long waves the $\psi^{(2)}$-field gives the dominating effect. Furthermore, for long waves the two contributions to M_{z1}, one from $\psi^{(2)}$ and the other due to products of first-order quantities, are of opposite sign and therefore partly cancel each other.

In this example, as well as in the examples discussed later with freely floating bodies, we find that the first term in (84) is the most important one, giving about three-quarters of the total contribution for the restrained ship, and about two thirds of the contribution for the freely floating ship.

For sufficiently small values of KL, the $\psi^{(2)}$-contribution to the yaw moment (84) may be evaluated approximately by analytical means. Intending to apply strip theory,

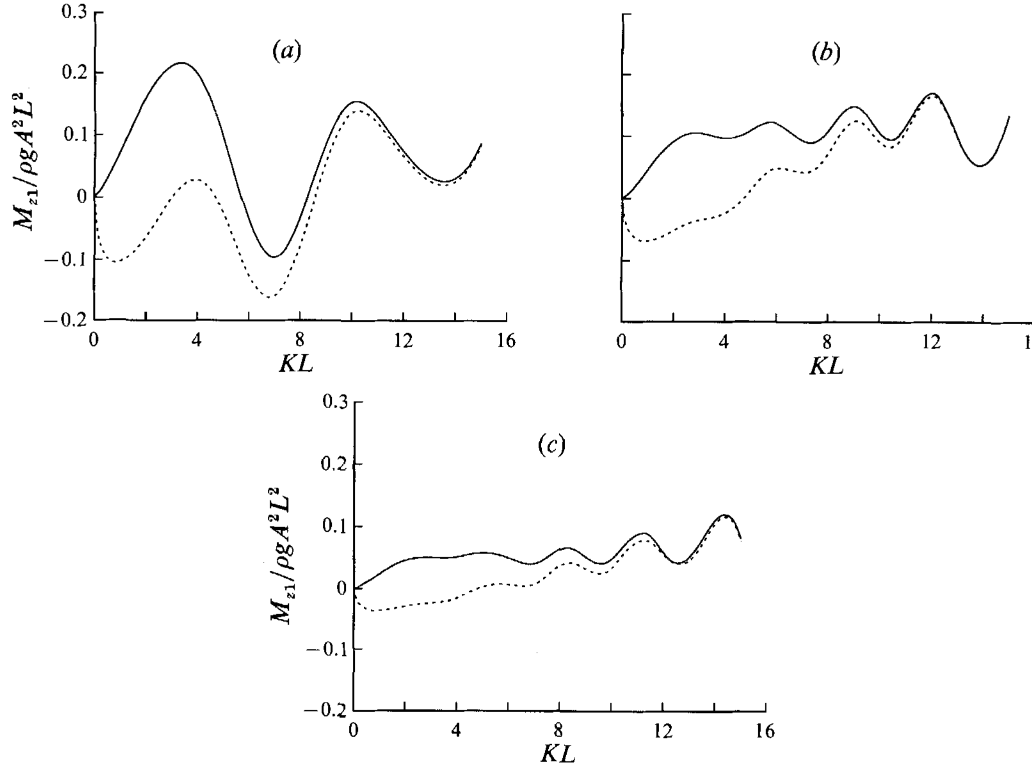

FIGURE 3. Forward speed part, M_{z1}, of mean yaw moment for the restrained ship ($L = 230$ m), obtained by the far-field method: solid line, total contribution; dotted line, without the $\psi^{(2)}$-contribution. (*a*) $\beta = 90°$, (*b*) $\beta = 140°$, (*c*) $\beta = 160°$.

we replace the wetted ship hull by a half circular cylinder of radius $\frac{1}{2}B$. Let ϕ_7 denote the velocity potential for the reflected wave; ϕ_7 then satisfies the boundary conditions

$$\frac{\partial \phi_7}{\partial n} = -\frac{\partial \phi_0}{\partial n} = -\frac{\partial \phi_0}{\partial y} n_2 - \frac{\partial \phi_0}{\partial z} n_3 \quad \text{on} \quad S_B \tag{86}$$

and

$$\frac{\partial \phi_7}{\partial z} = 0 \quad \text{on} \quad z = 0, \tag{87}$$

where $\partial\phi_0/\partial y$ and $\partial \phi_0/\partial z$ are constants, and n_2 and n_3 are the y- and z-coordinates of the normal vector $\boldsymbol{n}$. Comparing with (63)–(66), we may then write

$$\varphi \equiv \phi_0 + \phi_7 = \phi_0(1 + \mathrm{i}K \sin\beta \Psi - KZ) + O(|Kx|^2), \tag{88}$$

where $Z = Z(y, z)$ is a real function satisfying the two-dimensional Laplace equation, $\partial Z/\partial z = 0$ on S_F and $\partial Z/\partial n = n_3$ on S_B. To leading order Ψ is given by

$$\Psi = -B^2 y/4r^2, \tag{89}$$

where $r^2 = y^2 + z^2$. We then obtain

$$\mathrm{Im}\left(\varphi \frac{\partial^2 \varphi^*}{\partial z^2}\right) = -\frac{KB^2}{2y^3}\sin\beta + O(|Kx|^2)) \quad \text{at} \quad z = 0. \tag{90}$$

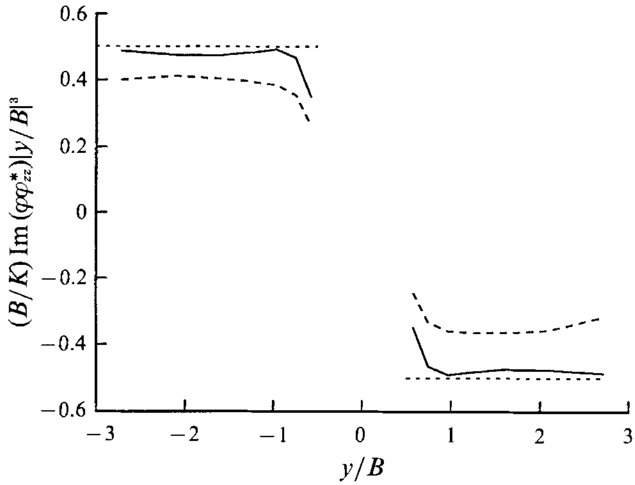

FIGURE 4. Values of $(B/K)\,\mathrm{Im}(\varphi\varphi^*_{zz})|y/B|^3$ *vs.* y/B, the lateral coordinate, obtained for the ship at the midsection. The midsection forms a rectangle with draught $T = 15$ m and beam $B = 41$ m. Solid line, $KB = 0.1833$; dashed line, $KB = 0.5092$; dotted line, analytical result for a half circular cylinder with radius $\frac{1}{2}B$.

Hence, from (47), we see that $\partial\psi^{(2)}/\partial z$ on S_F is antisymmetric with respect to y and decays as y^{-3}. In figure 4 the values of $\mathrm{Im}(\varphi\varphi^*_{zz})|y^3|$ are shown as function of y/B, obtained at the midsection of the ship, which is a rectangle form with draught $T = 15$ m and beam $B = 41$ m. The results for the ship are obtained by numerical computations. The angle of incidence is 90°, and $KB = 0.1833$ or 0.5092. Note the very good agreement between the analytical and numerical results for $KB = 0.1833$ for y-values not very close to the hull. But for $KB = 0.5092$ also the analytical result gives a fair approximation to the numerical result.

We may now evaluate the $\psi^{(2)}$-contribution to the yaw moment by using the above analysis for the half circular cylinder. Using the strip theory approximation and assuming that the ship's section is a half circular cylinder with beam being a function of the x-coordinate, $B(x) = B_0(1-|2x/L|^3)$ for $|x| < \frac{1}{2}L$, we find that

$$\frac{\tau}{2KL^2}\int_{S_F}\Psi_s\,\mathrm{Im}\left(\varphi\frac{\partial^2\varphi^*}{\partial z^2}\right)\mathrm{d}S = \tau\sin\beta\frac{B_0}{L}. \tag{91}$$

By introducing the same beam to length ratio as for the ship, the right-hand side of (91) becomes $0.178\tau\sin\beta$. Correspondingly, we find numerically for the ship that

$$\frac{\tau}{2KL^2}\int_{S_F}\Psi_s\,\mathrm{Im}\left(\varphi\frac{\partial^2\varphi^*}{\partial z^2}\right)\mathrm{d}S = 0.175\tau\sin\beta,$$

when $KB \ll 1$, which again illustrates the close agreement between the analytical and the numerical methods.

7.2. *The far-field method; non-restrained bodies*

It is expected that if the body is allowed to move, the $\psi^{(2)}$-contribution to the yaw moment may be changed considerably. That change will be most drastic for bodies with vertical walls extending deeply in the fluid (see §5.1). The motion of the body makes the solution of the problem more time-consuming. This is true for the first-order

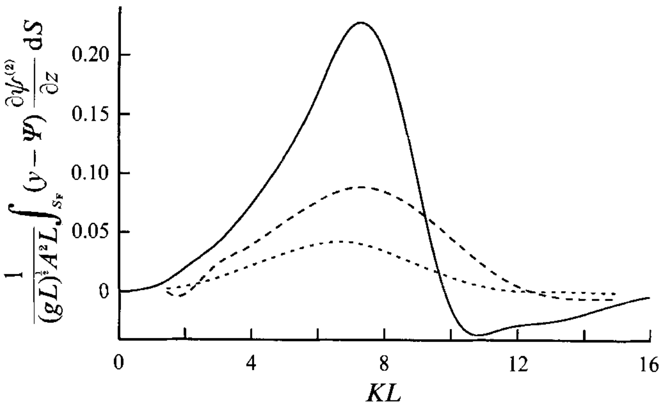

FIGURE 5. Non-dimensional values of $\int_{S_F}(y-\Psi)\,\partial\psi^{(2)}/\partial z\,\mathrm{d}S$ for the non-restrained ship (L = 230 m) *vs.* non-dimensional wavenumber KL: solid line, $\beta = 90°$; dashed line, $\beta = 140°$; dotted line, $\beta = 160°$.

problem; but the problem of obtaining $\psi^{(2)}$ when the first-order motion is known, is also changed. Now the problem is double-inhomogenous since the boundary conditions at the free surface as well as at the body are inhomogenous. In order to get an idea of the effect of a non-restrained body, the body is allowed to move freely to first order whereas we disregard its second-order motion. The boundary condition for $\psi^{(2)}$ at the body is then $\partial\psi^{(2)}/\partial n = 0$.

We have evaluated the $\psi^{(2)}$-contribution for the same ship as in the previous section. In figure 5 the sum of the two terms in (84) is shown as a function of KL for 90°, 140° and 160°. Comparing with figure 2 we observe that for restrained as well as for non-restrained structures the $\psi^{(2)}$-contribution is largest for waves with incidence angle equal to 90°. However, the main, and more interesting, difference between the two figures is that the maximal $\psi^{(2)}$-contributions are found for considerably larger values of KL ($4 < KL < 8$) in the non-restrained case than in the restrained case. This wavenumber interval corresponds to the wave period being larger than about 11 s and less than about 15 s, which is the interval where very often the wave spectrum has its maximal value.

Figure 6(*a*–*c*) shows the numerical results for the total value of M_{z1} and the value without the $\psi^{(2)}$-contribution, as a function of KL. Comparing figures 3(*a*) and 6(*a*) we notice that if the structure is allowed to move and the wave angle is 90°, the maximum value of M_{z1} is slightly increased. The main effect of the body being non-restrained is, however, that the $\psi^{(2)}$-contribution becomes important for more moderate wavelengths. When the wave angle is either 140° or 160° we observe that the maximum value of M_{z1} for the non-restrained case is 3–4 times larger than the maximal values for the restrained case. The maximum values of M_{z1} occur for moderate wavelengths for the non-restrained ship.

It is of interest to compare M_{z1} with the zero-speed moment M_{z0} for the ship, which is shown in figure 7 for $\beta = 140°$. By comparing figures 6(*b*) and 7 we observe that M_{z1} is about 25 times larger than M_{z0}. Thus, for $Fr \approx 0.02$, which for $L = 230$ m corresponds to $U = 1\ \mathrm{ms}^{-1}$, we find that the total moment is 50% larger than at zero speed.

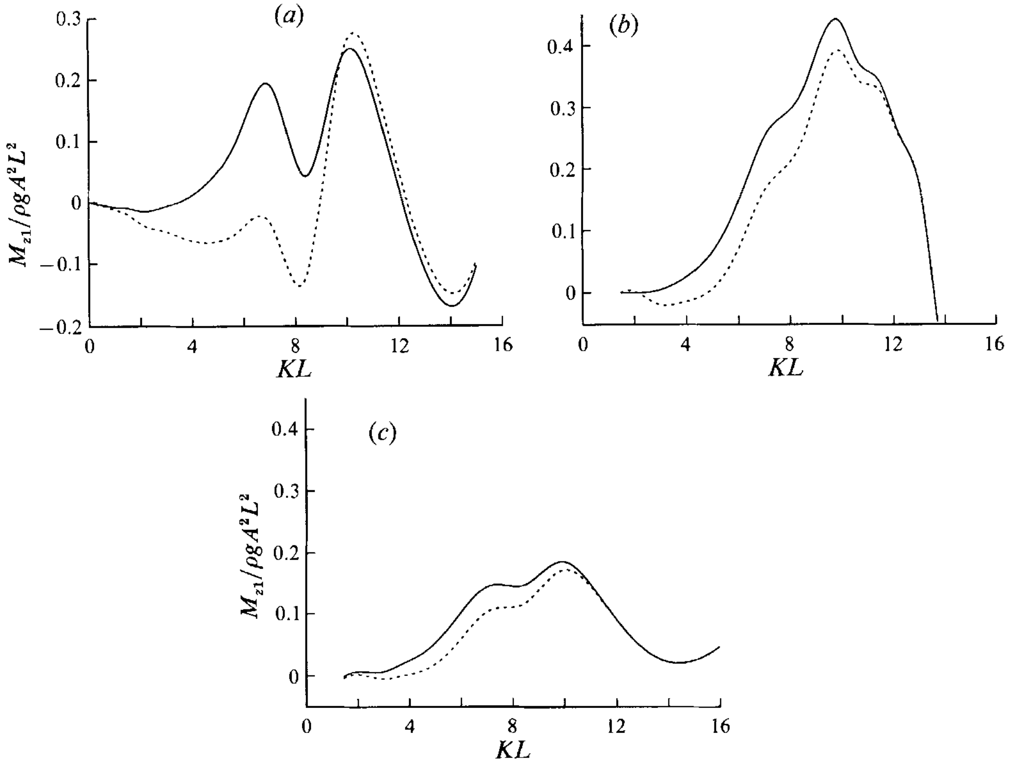

FIGURE 6. Forward speed part, M_{z1}, of mean yaw moment for the non-restrained ship ($L = 230$ m), obtained by the far-field method: solid line, total contribution; dotted line, without the $\psi^{(2)}$-contribution. (a) $\beta = 90°$, (b) $\beta = 140°$, (c) $\beta = 160°$.

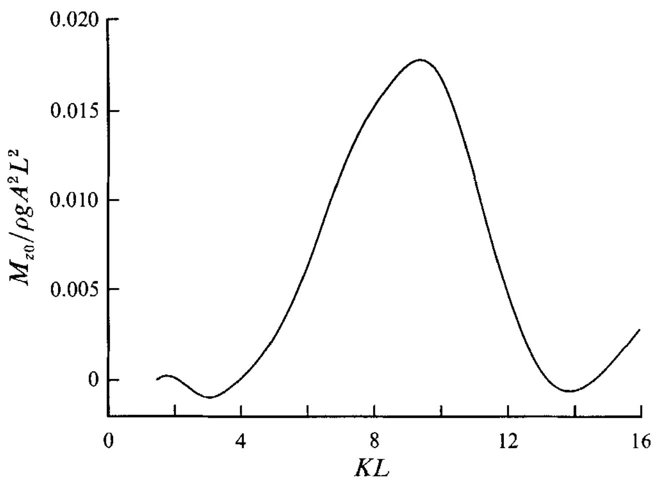

FIGURE 7. Mean yaw moment for the ship at zero forward speed, M_{z0}, obtained by the far-field method. Wave angle $\beta = 140°$.

7.3. *The near-field method*

If pressure integration is used, the terms

$$-\rho U \int_{S_B} \nabla \chi_s \cdot \nabla \psi^{(2)} n_i \,\mathrm{d}S, \quad i = 1, 2, \ldots 6$$

in (5) appear. The $\psi^{(2)}$-contribution to the mean yaw moment is, according to (7), given by

$$-\rho U \int_{S_F+S_B} \Psi \frac{\partial \psi^{(2)}}{\partial z} \mathrm{d}S - \rho U \int_{S_F+S_B} \frac{\partial \psi^{(2)}}{\partial z} \left(x \frac{\partial \chi}{\partial y} - y \frac{\partial \chi}{\partial x} \right) \mathrm{d}S. \tag{92}$$

The first term is identical to the second term of (84). In the present examples this term is found to give the dominant contribution to (92), since the last term in all examples is about 25% of the first term. As noted above, the second term of (84) contributes approximately one third of the total contribution from (84) in the far-field case (non-restrained bodies). Thus, the magnitude of (92) is about one third of (84) in the present examples. Therefore, if the $\psi^{(2)}$-contribution is totally neglected, it is a better approximation to use the near-field method than the far-field method.

The contribution from $\psi^{(2)}$ to the horizontal forces by the near-field method is, according to (6), given by

$$-\rho U \int_{S_F+S_B} \frac{\partial \psi^{(2)}}{\partial z} \frac{\partial \chi}{\partial x_i} \mathrm{d}S, \quad i = 1, 2. \tag{93}$$

Like the last term in (92) this term is found to be non-zero, but small. We have performed several simulations, not reported here, for different body geometries freely oscillating in the incoming waves. Even when resonant body motions occur, and the contribution from the evanescent modes is significant, we find that (93) and the last term of (92) contribute at most 5% to the forward speed part of respectively the horizontal forces and yaw moment. The $\psi^{(2)}$-contribution to the yaw moment, obtained either by the near- or far-field method is, however, not small and needs to be accounted for in practical applications.

8. Summary and conclusion

The effect of the steady second-order potential $\psi^{(2)}$ on the drift forces and moments experienced by a floating or submerged body in waves and a current is discussed. This contribution seems to have been neglected in the literature. The problem of evaluating $\psi^{(2)}$ becomes complicated for bodies of arbitrary form moving with a finite speed, since the vertical displacement of the free surface is finite. For small U the problem reduces to solving a linear integral equation. We show, however, that the $\psi^{(2)}$-contributions to the horizontal drift forces by the near-field method, and to the yaw moment by the near-field and far-field method can be obtained without solving for $\psi^{(2)}$. For the horizontal drift forces obtained by the far-field method it is found that $\psi^{(2)}$ does not contribute, provided that there is no velocity circulation in the fluid.

The $\psi^{(2)}$-field, evaluated for zero speed, is found to originate from first-order evanescent modes and linear body responses. $\psi^{(2)}$ is therefore zero (or constant) when the body is restrained and has vertical walls extending deeply in the fluid. Numerical results show that the effect of $\psi^{(2)}$ is significant to the yaw drift moment. We find that the most important contribution occurs where the wave spectrum often has its maximal value.

The contribution to the horizontal forces from $\psi^{(2)}$, which is identically zero using the far-field method, is found to be non-zero when the near-field method is applied. It seems however, after several simulations for different geometries, that this contribution is at most 5% of the forward speed part of the forces. In practical applications the $\psi^{(2)}$-term may then be neglected when evaluating the horizontal forces by the near-field method.

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Appendix. Contributions to the moment due to the first-order potential, for small U

Contributions to the mean yaw moment due to the first-order potential are given by

$$M_z = -\rho\int_{S_\infty} \overline{\frac{\partial\phi^{(1)}}{\partial\theta}\frac{\partial\phi^{(1)}}{\partial R}}\,\mathrm{d}S - \rho U\int_{C_\infty} \overline{\left(y\frac{\partial\phi^{(1)}}{\partial R}-\frac{x}{R}\frac{\partial\phi^{(1)}}{\partial\theta}\right)\zeta^{(1)}}\,\mathrm{d}s. \tag{A 1}$$

Introducing $\phi^{(1)} = \mathrm{Re}(\phi \mathrm{e}^{\mathrm{i}\sigma t})$ and averaging in time we obtain

$$\frac{2}{\rho}M_z = \int_0^{2\pi}\left\{-R\int_{-\infty}^0 \mathrm{Re}\left[\frac{\partial\phi}{\partial\theta}\frac{\partial\phi^*}{\partial R}\right]\mathrm{d}z - \tau R^2\sin\theta\,\mathrm{Re}\left[\mathrm{i}\frac{\partial\phi}{\partial R}\phi^*\right]_{z=0} + \tau R\cos\theta\,\mathrm{Re}\left[\mathrm{i}\frac{\partial\phi}{\partial\theta}\phi^*\right]_{z=0}\right\}\mathrm{d}\theta. \tag{A 2}$$

The potential ϕ is composed of $\phi = A(\mathrm{i}g/\omega)(\phi_0+\phi_\mathrm{B})$, see (33), (35) and (36).

To obtain contributions due to ϕ_B only we introduce

$$\frac{\partial\phi_\mathrm{B}}{\partial\theta} = \left((z-\mathrm{i}R)\frac{\partial k_1}{\partial\theta}+\frac{\partial H/\partial\theta}{H}\right)\phi_\mathrm{B} + O(R^{-\frac{3}{2}}, R\tau^2, \tau^2), \tag{A 3}$$

where

$$\partial k_1/\partial\theta = -2\nu\tau\sin\theta + O(\tau^2), \tag{A 4}$$

into the first term of (A 2). Carrying out the vertical integration we obtain

$$-R\int_{-\infty}^0 \mathrm{Re}\left[\frac{\partial\phi_\mathrm{B}}{\partial\theta}\frac{\partial\phi_\mathrm{B}^*}{\partial R}\right]\mathrm{d}z = -R\frac{\nu\tau}{2k_1^2}\sin\theta\,\mathrm{Re}\left[\phi_\mathrm{B}\frac{\partial\phi_\mathrm{B}^*}{\partial R}\right]_{z=0} + R^2\frac{\nu\tau}{k_1}\sin\theta\,\mathrm{Re}\left[\mathrm{i}\frac{\partial\phi_\mathrm{B}}{\partial R}\phi_\mathrm{B}^*\right]_{z=0} - \frac{R}{2k_1}\mathrm{Re}\left[\frac{\partial H/\partial\theta}{H}\phi_\mathrm{B}\frac{\partial\phi_\mathrm{B}^*}{\partial R}\right]_{z=0} + O(R^{-1}, R\tau^2, \tau^2). \tag{A 5}$$

We then introduce (A 5) into (A 2), and apply that

$$\partial\phi_\mathrm{B}/\partial R = (-\mathrm{i}k_1 - \tfrac{1}{2}R^{-1})\phi_\mathrm{B} + O(R^{-\frac{5}{2}}, R\tau^2, \tau^2). \tag{A 6}$$

Noting that $\mathrm{Re}[\phi_\mathrm{B}\,\partial\phi_\mathrm{B}^*/\partial R] = O(R^{-2})$ we find that the contributions to the moment due to ϕ_B only, to leading order in τ, become

$$\frac{M_z^{(\mathrm{B})}}{\rho g^2A^2/2\omega^2} = \tau R\int_0^{2\pi}\mathrm{d}\theta\cos\theta\,\mathrm{Re}\left[\mathrm{i}\frac{\partial\phi_\mathrm{B}}{\partial\theta}\phi_\mathrm{B}^*\right]_{z=0} - R\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{Re}\left[\frac{\partial H/\partial\theta}{2k_1H}\phi_\mathrm{B}\frac{\partial\phi_\mathrm{B}^*}{\partial R}\right]_{z=0} = -\frac{1}{2}\int_0^{2\pi}(1-2\tau\cos\theta)\,\mathrm{Im}\left[H\frac{\partial H^*}{\partial\theta}\right]\mathrm{d}\theta. \tag{A 7}$$

To obtain contributions due to products between ϕ_0 and ϕ_B, the first term of (A 2) gives

$$I_1 = -\int_{-\infty}^{0}\int_{0}^{2\pi} \mathrm{Re}\left[\frac{\partial\phi_0}{\partial\theta}\frac{\partial\phi_B^*}{\partial R}+\frac{\partial\phi_B}{\partial\theta}\frac{\partial\phi_0^*}{\partial R}\right] R\,\mathrm{d}\theta\,\mathrm{d}z$$

$$= -\int_{-\infty}^{0}\int_{0}^{2\pi} \mathrm{Re}\left[-\phi_0\frac{\partial^2\phi_B^*}{\partial R\,\partial\theta}+\frac{\partial\phi_B}{\partial\theta}\frac{\partial\phi_0^*}{\partial R}\right] R\,\mathrm{d}\theta\,\mathrm{d}z, \qquad \text{(A 8)}$$

where in the first term we have applied partial integration in the θ-variable. By introducing

$$\frac{\partial^2\phi_B}{\partial R\,\partial\theta} = \left[\left(-\mathrm{i}k_1 z - k_1 R - \frac{\mathrm{i}}{2}\right)\frac{\partial k_1}{\partial\theta} - \mathrm{i}k_1\frac{\partial H/\partial\theta}{H}\right]\phi_B + O(R^{-\frac{3}{2}}, R\tau^2, \tau^2) \qquad \text{(A 9)}$$

into (A 8) and carrying out the vertical integration, neglecting terms $O(R^{-1}, R\tau^2, \tau^2)$, we obtain

$$I_1 = \tau\int_0^{2\pi}\left\{KR^2\sin\theta(1+\cos(\beta-\theta))\,\mathrm{Re}[\phi_0\phi_B^*]_{z=0} + \frac{R}{2}\sin\theta\cos(\beta-\theta)\,\mathrm{Re}[\mathrm{i}\phi_0\phi_B^*]_{z=0}\right\}\mathrm{d}\theta$$

$$+R\int_0^{2\pi}\frac{k_1+K\cos(\beta-\theta)}{k_1+K}\,\mathrm{Re}\left[\mathrm{i}\frac{\partial H^*/\partial\theta}{H^*}\phi_0\phi_B^*\right]_{z=0}\mathrm{d}\theta. \qquad \text{(A 10)}$$

The second term of (A 2) becomes

$$I_2 = -\tau R^2\int_0^{2\pi}\sin\theta\,\mathrm{Re}\left[\mathrm{i}\frac{\partial\phi_0}{\partial R}\phi_B^* + \mathrm{i}\frac{\partial\phi_B}{\partial R}\phi_0^*\right]_{z=0}\mathrm{d}\theta$$

$$= -\tau\int_0^{2\pi}\left\{KR^2\sin\theta\left(\frac{k_1}{K}+\cos(\beta-\theta)\right)\mathrm{Re}[\phi_0\phi_B^*]_{z=0} + \frac{R}{2}\sin\theta\,\mathrm{Re}[\mathrm{i}\phi_0\phi_B^*]_{z=0}\right\}\mathrm{d}\theta, \qquad \text{(A 11)}$$

and for the third term of (A 2) we have

$$I_3 = \tau R\int_0^{2\pi}\cos\theta\,\mathrm{Re}\left[\mathrm{i}\frac{\partial\phi_0}{\partial\theta}\phi_B^* + \frac{\partial\phi_B}{\partial\theta}\phi_0^*\right]_{z=0}\mathrm{d}\theta$$

$$= \tau R\int_0^{2\pi}\sin\theta\,\mathrm{Re}[\mathrm{i}\phi_0\phi_B^*]_{z=0}\,\mathrm{d}\theta - 2\tau R\int_0^{2\pi}\cos\theta\,\mathrm{Re}\left[\mathrm{i}\phi_0\frac{\partial\phi_B^*}{\partial\theta}\right]_{z=0}\mathrm{d}\theta, \qquad \text{(A 12)}$$

where we have applied partial integration. The three contributions added up give then, omitting $O(R^{-1}, R\tau^2, \tau^2)$,

$$I_1+I_2+I_3 = R\int_0^{2\pi}\left(\frac{k_1+K\cos(\beta-\theta)}{k_1+K} - 2\tau\cos\theta\right)\mathrm{Re}\left[i\frac{\partial H^*/\partial\theta}{H^*}\phi_0\phi_B^*\right]_{z=0}\mathrm{d}\theta$$

$$+\frac{\tau R}{2}\int_0^{2\pi}\sin\theta(1+\cos(\beta-\theta))\,\mathrm{Re}[\mathrm{i}\phi_0\phi_B^*]_{z=0}\,\mathrm{d}\theta. \qquad \text{(A 13)}$$

Introducing (35) and (36), and applying the method of stationary phase to the integrals, we obtain for the total moment due to the first-order velocities

$$\frac{M_z}{\rho g A^2} = -\frac{1}{4K}\mathrm{Im}\left\{\int_0^{2\pi}(1-2\tau\cos\theta)\,H\frac{\mathrm{d}H^*}{\mathrm{d}\theta}\,\mathrm{d}\theta\right\} - \frac{1}{2K}\mathrm{Im}\left\{\frac{\nu}{K}S' + \tau\sin\beta S\right\} + O(\tau^2),$$

(A 14)

where $$S = (2\pi/\nu)^{\frac{1}{2}} e^{i\pi/4} H^*(\beta + 2\tau \sin\beta) \tag{A 15}$$

and $$S' = (2\pi/\nu)^{\frac{1}{2}} e^{i\pi/4} (H'(\beta + 2\tau \sin\beta))^*. \tag{A 16}$$

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